EXPLANATION OF MYP TASK TWO

WRITTEN REPORT OF AN INVESTIGATION

Students wrote this in class under test conditions. They had to email their work at the end of class each day. They had a total of 2.5 hours to complete the tasks and were given the reflection to take home and hand in the next day.

Marking Scheme

- All MYP criterion based assessment, focusing on:
    - Criterion B: Investigating Patterns
    - Criterion C: Communication in Mathematics
    - Criterion D: Reflection in Mathematics.

Criterion B: Investigating Patterns

- Students are to determine the general rules associated with transformations of quadratic functions
- Open-ended to allow all levels of ability – students who are more capable can combine all transformations and look at more advanced functions, such as cubics, to see if the same general rules they found to exist for parabolas, are true for higher order functions
- They are asked to validate their generalizations and apply them to a particular question to ensure they are correct in all cases
- Students are asked to justify/validate their general rule that they have determined.

Criterion C: Communication in Mathematics

- Students are asked to summarize their findings and general rules in a one page report, illustrating all of the key information, using correct symbols and terminology
- Students must show evidence of moving between the algebraic and graphic forms of quadratic functions.

Criterion D: Reflection in Mathematics

- Students are asked to critically explain the results obtained in the context of the problem.
- Students are asked when to apply their general rule, why it is presented in that particular form and how they can connect their findings to real life.
- Students were asked to suggest improvements to their problem-solving methodology.
- They are also asked to reflect on the use of technology, which was instrumental in allowing them to focus on determining the rule.

Part Two: Summary of generalizations

Using all of the information you discovered in Part One, create a one to two page summary of all of the different types of transformations, detailing your generalizations about each type and a diagram to illustrate your general rules.

Since you are not handing in the answers to part one separately, you must make sure that you include all of the necessary points that you determined in your answers to the questions in part one.

This is part of your communication mark, so make sure you clearly explain your reasoning, using appropriate terminology and symbols.

All of your diagrams are to be created using graphing software

Question five: Translations

$y = x^2$

$y = (x-1)^2$

$y = (x+3)^2$

(a) Graph the above functions on the same set of axes

(b) Copy and paste the above functions in your Word document.

(c) What is the axis of symmetry and vertex of each function?

(d) What do the graphs have in common?

(e) What effect does h have on the graph $y = (x - h)^2$ if $h > 0$?

(f) What effect does h have on the graph $y = (x - h)^2$ if $h < 0$?

(g) Write a general statement about the effect of changes in h on the graph $y = x^2$

Question six: Combinations

$y = x^2$

$y = -2(x-4)^2 + 3$

(a) Graph the above functions on the same set of axes

(b) Copy and paste the above functions in your Word document.

(c) What is the axis of symmetry and vertex of each function?

(d) How do the constraints -2, 4 and 3 affect the graph $y = x^2$ (do a step by step transformation from $y = x^2$ to $y = -2(x - 4)^2 + 3$)

(e) Describe, in general, what effect each constraint (a, h and k) has on the graph $y = a(x - h)^2 + k$ compared to $y = x^2$?

Investigate how a, h and k affect the graph of $y = x^2$ in the equation of $y = a(x - h)^2 + k$.

Part One: Transformations of Quadratic Functions

Question one: Dilations

$y = x^2$

$y = 2x^2$

$y = 4x^2$

(a) Graph the above functions on the same set of axes.

(b) Copy and paste the above functions in your Word document.

(c) What is the axis of symmetry and vertex of each function?

(d) What do the graphs have in common?

(e) What happens to the graph of $y = x^2$ when x is multiplied by a positive number greater than 1?

(f) Write a general statement about the effect of changes in *a* on the graph $y = ax^2$ when $a > 1$?

Question two: Dilations

$y = x^2$

$y = \frac{1}{2}x^2$

$y = \frac{1}{4}x^2$

(a) Graph the above functions on the same set of axes:

(b) Copy and paste the above functions in your Word document.

(c) What is the axis of symmetry and vertex of each function?

(d) What do the graphs have in common?

(e) What happens to the graph of $y = x^2$ is multiplied by a positive number less than 1 but larger than zero?

(f) Write a general statement about the effect of changes in a on the graph $y = ax^2$ when $1>a>0$?

Question three: Reflections

$y = x^2$

$y = -2x^2$

$y = -\frac{1}{3}x^2$

(a) Graph the above functions on the same set of axes:

(b) Copy and paste the above functions in your Word document.

(c) What is the axis of symmetry and vertex of each function?

(d) What do the graphs have in common?

(e) What happens to the graph of $y = x^2$ is multiplied by a negative number?

(f) Write a general statement about the effect of changes in a on the graph $y = ax^2$ when $a<0$?

Question four: Translations

$y = x^2$

$y = x^2 + 2$

$y = x^2 - 4$

(a) Graph the above functions on the same set of axes

(b) Copy and paste the above functions in your Word document.

(c) What is the axis of symmetry and vertex of each function?

(d) What do the graphs have in common?

(e) What effect does k have on the graph $y = x^2 + k$ if $k > 0$?

(f) What effect does k have on the graph $y = x^2 + k$ if $k < 0$?

(g) Write a general statement about the effect of changes in k on the graph $y = x^2$

Investigate how a, h and k affect the graph of $y = x^2$

in the equation of $y = a(x - h)^2 + k$.

Part Three: Examples to validate your general rules

Create an illustration of each of the following parabolas, using $y = x^2$ as a reference on each illustration.

Clearly explain **each step** that is required to transform the parabola $y = x^2$ into each of the following 4 separate parabolas.

For each, clearly label:

a) the axis of symmetry (equation of the line)

b) the vertex

c) the domain and range

$$y = -\frac{1}{4}x^2$$

$$y = 2x^2 - 10$$

$$y = -4(x+3)^2$$

$$y = \tfrac{1}{2}(x-2)^2 + 8$$

Part Four: Reasoning and Justification

Answer the following questions in sentence form, using examples where suitable to illustrate your point:

How did the use of technology help you during the course of this investigation?

Can you think of any improvements that you could have made to your problem solving methodology during this assignment?

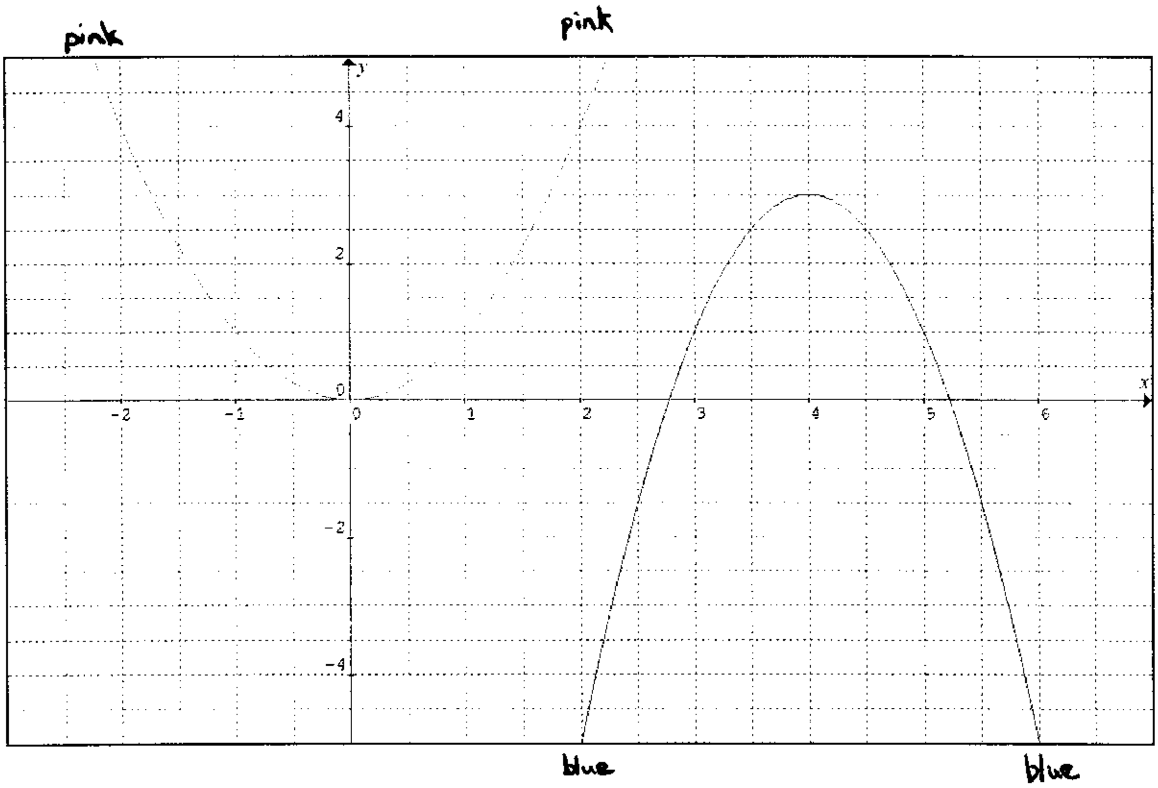
Why is the generalized form of the equation expressed as $y = a(x-h)^2 + k$?

How do you think this formula could be used in real life applications?

How do you know your generalizations about transformations are correct? Justify/validate your general rules.

Can these generalizations be applied to other functions? Explain your reasoning.

ANSWER KEY TO TRANSFORMATIONS ASSIGNMENT



The pink function is $y = x^2$ and the blue function is $y = -2(x-4)^2 + 3$

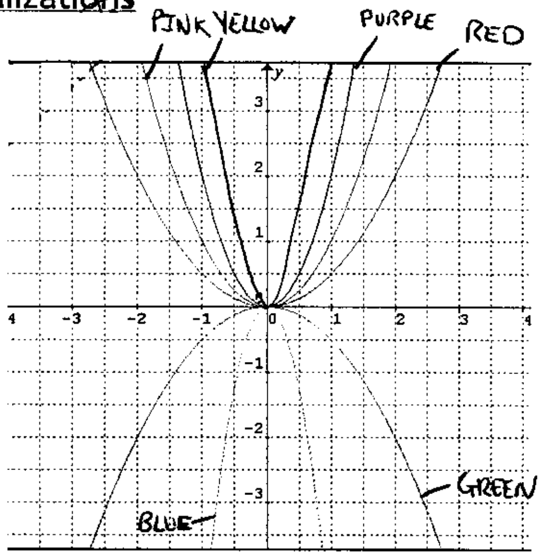
Transformations that have taken place to the basic $y = x^2$ to get $y = -2(x-4)^2 + 3$

transformation	number	explanation
dilation	2	- proper mathematical terminology is a vertical stretch - would accept *y* value increases at a faster rate than it did in the original function - in this case, the function has stretched vertically by a factor of 2
reflection	Negative in front of the 2	- reflection in the x axis - causes the function to open downwards instead of upwards - function now has a maximum instead of a minimum - in this case, the function has been reflected in the x axis
translation	4	- horizontal shift - affects the *x* co-ordinate of the vertex - in this case, the function has shifted 4 units to the right and 4 in the *x* co-ordinate of the vertex
translation	3	- vertical shift - affects the *y* co-ordinate of the vertex - in this case, the function has shifted 3 units up and 3 is the *y* co-ordinate of the vertex

Transformations that have taken place to the basic $y = x^2$ to get $y = a(x-h)^2 + k$

transformation	variable	explanation
dilation	*a*	When *a* is greater than 1, the function is stretched vertically (*y* value increases at a faster rate than it did in the original function). When *a* is less than 1 but greater than 0, the function is compressed vertically (*y* value increases at a slower rate than it did in the original function).
reflection	*-a*	When the *a* value is negative, the function is reflected in the *x* axis, which means the function opens downwards and has a maximum value.
translation	*h*	A change in *h* causes a horizontal shift (movement on the x axis). When *h* is positive it shifts to the right and when *h* is negative it shifts to the left. The shape of the parabola does not change. A horizontal shift affects the vertex of the function and the *h* value represents the *x* co-ordinate of the vertex of the equation.
translation	*k*	A change in *k* causes a vertical shift (movement on the y axis). When *k* is positive it shifts up and when *k* is negative it shifts down. The shape of the parabola does not change. A vertical shift affects the vertex of the function and the k value represents the *y* co-ordinate of the vertex of the equation.

Part 2: Summary of Generalizations



(RED) $y = \frac{1}{2}x^2$ $y = x^2$ (PINK)

(GREEN) $y = -\frac{1}{2}x^2$ $y = 4x^2$ (YELLOW)

(PURPLE) $y = 2x^2$ $y = -5x^2$ (BLUE)

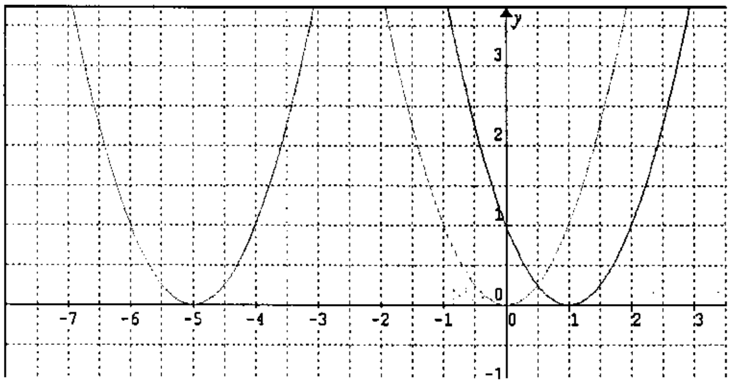
Changing the value of a :

Within quadratics, the general observations that I have made, suggests that once the value of *a* is greater, or equal to one and the numbers increase, the larger the number the narrower the opening of the parabola becomes. For example, on the diagram on the left, the red, magenta and dark blue parabolas are each wider than that of the yellow one. They each have the same vertex and *h* value, as well as the same minimum value and line of symmetry however they each have different *a* values. The yellow parabola has the highest *a* value, while the red parabola has the lowest *a* value. This proves the idea that as the a value decreases, the parabola's opening gets wider. The other name used in this case would be 'dilation'

However, in the cases where *a* is less than one and bigger than zero, the closer the number is to zero, the larger the slope of the parabola become. Again the value of *a* does not have an effect on the line of symmetry, the vertex or the range. We can see as well that the red line on the graph has a minimum value.

When the value for *a* is a negative and therefore less than zero, when the numbers decrease in value the slope of the parabola would decrease. This is seen in the graph, as the dark green line, which has a lower value, is steeper than the light blue line, which has a higher value of *a*.

As well, when the value for *a* is less than zero, the range changes, as now the line gets a maximum value. The parabola therefore looks as if it has been flipped. However, the line of symmetry and vertex don't differ from those of positive value...



$y = x^2$

$y = (x-1)^2$

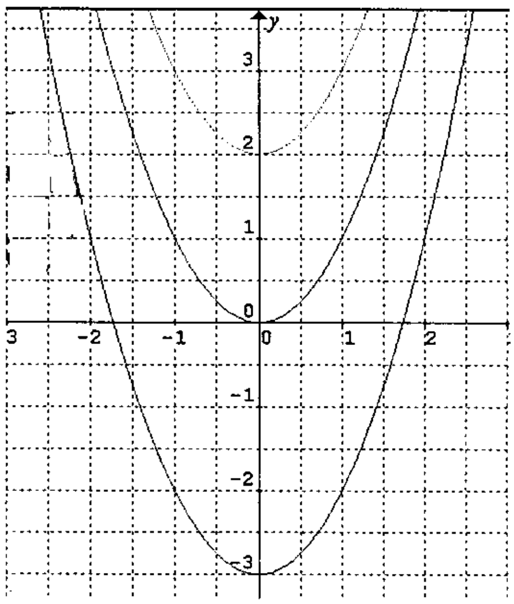
$y = (x+5)^2$

Changing the value of h :

When the value of *h* changes, it leads to the change in the line of symmetry. As well, it changes the value of the vertex. If *h* has a value that is bigger than zero, the line of symmetry will shift to the left of the x-axis, which can be shown with the red line. When h is smaller than zero the line of symmetry will shift to the right of the x-axis, as seen with the blue line. The line of symmetry becomes the value of h, with the opposite sign in front of it. This value of h, with the opposite sign in front of it, also becomes the x value for the vertex. The range is not changed and the line is always minimized. The change of *a*, does not affect the range, and the line holds a minimum value.

Changing the value for k :

When changing the value of *k*, it changes the y intercept of the line, and therefore the y value of the vertex. The *k* value is the y ~~intercept~~, and therefore the bigger it is the higher up the parabola is on the graph and y axis. This is seen in the graph on the right as the pink value of the y intercept is 2 and it's y intercept becomes 2. Additionally, the value of *k* in the red line is –3, and again the y intercept becomes –3. This is as the *k* value, changes the y value in the vertex, therefore shifting the whole vertex. As the y intercept is changed, the range is changed as well. However, the line of symmetry stays the same, as the x value in the vertex is not changed.



$y = x^2$

$y = x^2 + 2$

$y = x^2 - 3$

Investigate how a, h and k affect the graph of y = x²

in the equation of y = a(x – h)² + k.

Part Three: Examples to validate your general rules

Create an illustration of each of the following parabolas, using $y = x^2$ as a reference on each illustration.

Clearly explain **each step** that is required to transform the parabola $y = x^2$ into each of the following 4 separate parabolas.

For each, clearly label:

a) the axis of symmetry (equation of the line)

b) the vertex

c) the domain and range

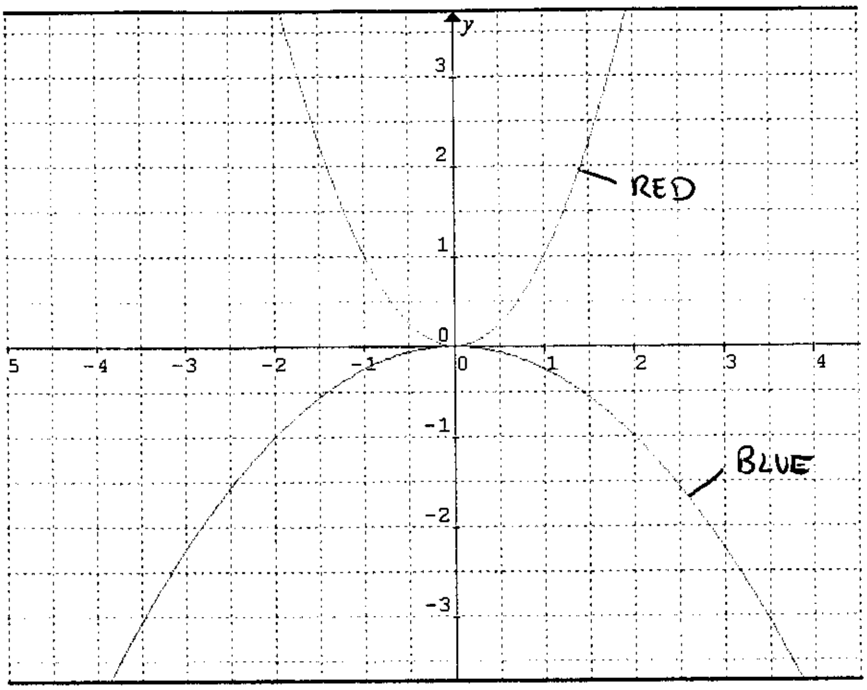
$$y = -\frac{1}{4}x^2$$

$$y = 2x^2 - 10$$

$$y = -4(x+3)^2$$

$$y = \tfrac{1}{2}(x-2)^2 + 8$$

1)



$y = x^2$ (RED)

$y = -\frac{1}{4}x^2$ (BLUE)

In order to transform the y=x^2 value on the graph, into the $y = -\frac{1}{4}x^2$ parabola, the *a* value of y=ax^2 must change. Because the value of a in the parabola y=x^2 is one, if changed to for example $\frac{1}{4}$, the parabola would be transformed into the mirror of the $y = -\frac{1}{4}x^2$ with the x-axis. To then put the parabola into a maximum value, we must change *a* into a negative number. When this is done, the parabola flips downwards and becomes the same line as the $y = -\frac{1}{4}x^2$.

a) the axis of symmetry (equation of the line)

The axis of symmetry is x=0

b) the vertex

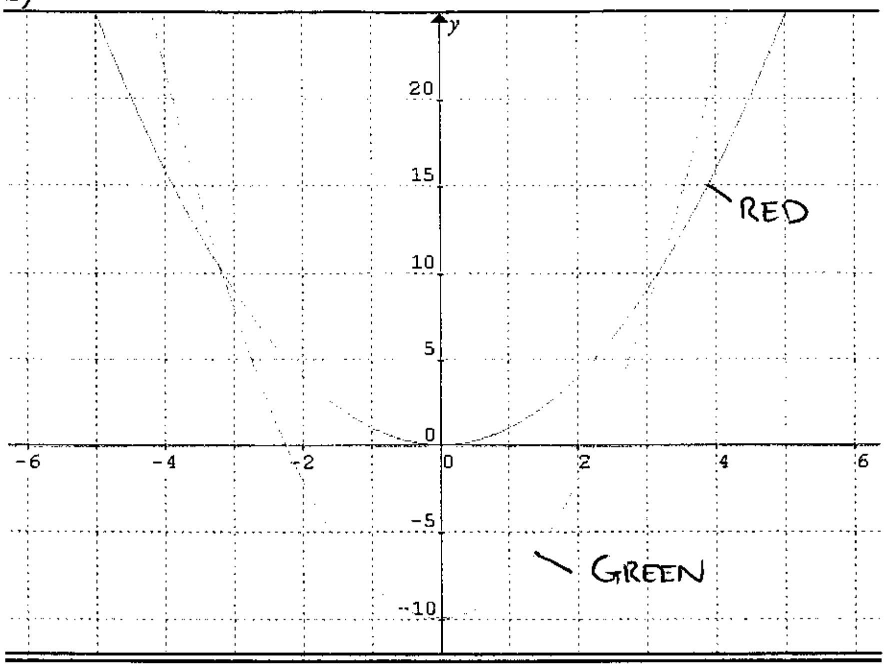
Both lines have the same axis of symmetry, as well as the same vertex which is point (0;0).

c) the domain and range

the domain of the parabola is $\{x / x \leq 0; x \in R\}$

the range of the parabola is $\{y / y \in R\}$

2)



$y = x^2$ (RED)

$y = 2x^2 - 10$
(GREEN)

In order to transform the y=x² value on the graph, into the $y = 2x^2 - 10$ parabola, the *k* and *a* value of y=ax² +k must change. Because the value of *a* in the parabola y=x² is one, if changed to for example 2, the parabola would be transformed into the same slope of the $y = 2x^2 - 10$ parabola. However this parabola would not be the same because of the *k* value. The *k* value of $y = 2x^2$ is zero, therefore in order to move the parabola down the y axis, it is necessary to change the *k* value to -10. Once this is done, the line overlaps and becomes the same line as the $y = 2x^2$.

a) the axis of symmetry (equation of the line)

The axis of symmetry is x=0

b) the vertex

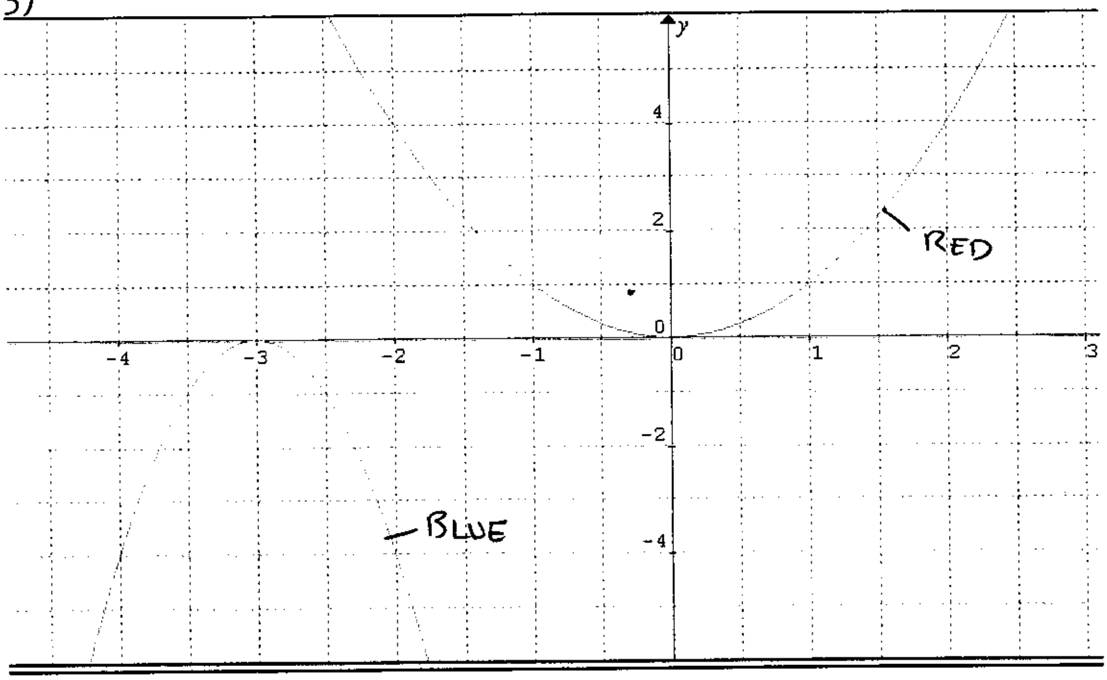
The vertex is (0;-10)

c) the domain and range

the domain of the parabola is $\{X / X \geq -10; x \in R\}$

the range of the parabola is $\{ Y / Y \in R\}$

3)



$y = x^2$ (RED)

$y = -4(x+3)$ (BLUE)

In order to transform the $y=x^2$ value on the graph, into the $y=-4(x+3)^2$ parabola, the *h* and *a* values of $y = a(x-h)^2$ must change. Because the value of a in the parabola $y=x^2$ is one, if changed to for example -4, the parabola would be transformed into the same slope of the $y=-4(x+3)^2$ parabola. However this parabola would not be the same because of the *h* value. The *h* value of $y=-4x^2$ is zero, therefore in order to move the parabola down the x axis, towards the left, it is necessary to change the *h* value to +3. Once this is done, the lines overlap each other and becomes the same line as the $y=-4(x+3)^2$.

a) the axis of symmetry (equation of the line)

The axis of symmetry is x= -3

b) the vertex

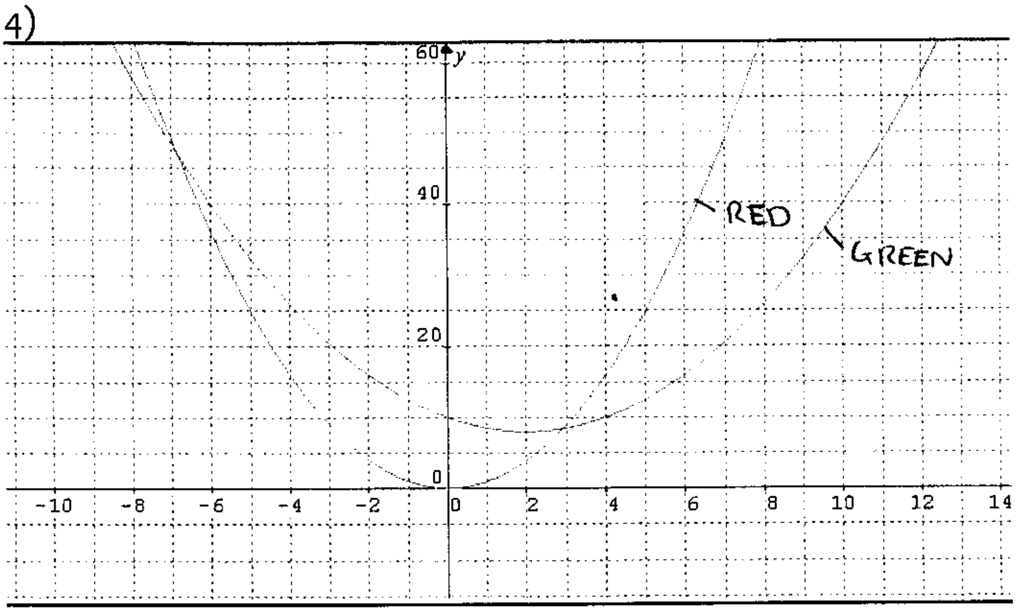
The vertex is (-3;+0)

c) the domain and range

the domain of the parabola is $\{X / X \leq 0; x \in R\}$

the range of the parabola is $\{ Y / Y \in R\}$

4)



$y = x^2$ (RED)

$y = \frac{1}{2}(x-2)^2 + 8$ (GREEN)

In order to transform the $y=x^2$ value on the graph, into the $y = \frac{1}{2}(x-2)^2 + 8$ parabola, the *h*, *k* and *a* values of $y = a(x-h)^2 + k$ must change. Because the value of a in the parabola $y=x^2$ is one, if changed to for example 1/2, the parabola would be transformed into the same slope of the $y = \frac{1}{2}(x-2)^2 + 8$ parabola. However this parabola would not be the same because of the *h* value. The *h* value of $y = \frac{1}{2}x^2$ is zero, therefore in order to move the parabola down the x axis, towards the right, it is necessary to change the *h* value to -2. The *k* value must also be changed in order for the parabola to move up the y axis, to +8 Once this is done, the lines overlap each other and becomes the same line as the $y = \frac{1}{2}(x-2)^2 + 8$.

a) the axis of symmetry (equation of the line)

The axis of symmetry is x= 2

b) the vertex

The vertex is (2;1.7)

c) the domain ~~and~~ range

the domain of the parabola is $\{x / x \geq 1.7 : x \in R\}$

the range of the parabola is $\{y / y \in R\}$

Part Four: Reasoning and Justification

Answer the following questions in sentence form, using examples where suitable to illustrate your point:

How did the use of technology help you during the course of this investigation?

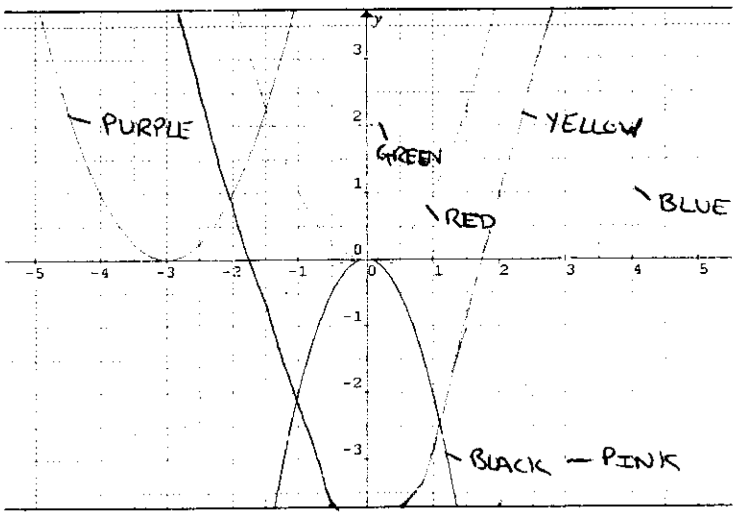
The use of technology helped me during this investigation because it helped to visualize the graph, rather than depending on my knowledge of parabolas and the *a*, *h* and *k* values and how they affect quadratics. I also used math software such as Graphmatica and Math type. They helped me during this investigation because they made it much easier to display.

Can you think of any improvements that you could have made to your problem solving methodology during this assignment?

A way that I could have improved my problem solving methodology would be to validate my calculations and answers however I did not have enough time. Another way of improving my method would be to use as many examples as possible to fully display and demonstrate my outlook.

Why is the generalized form of the equation expressed as $y = a(x-h)^2 + k$?

With the $y = a(x-h)^2 + k$ equation, each coefficient *a*, *k* and *h* have individual effects on the parabola. When each is altered or changed, it is clearly changed on the actual parabola. This can be seen with the red parabola $y=x^2$. It is the only equation that has values such as 0 or 1 for the *a*, *h* and *k* variables. Obviously, as viewed on the graph, the characteristics of each parabola change as the variables change.



$y = x^2$ (RED)

$y = \frac{1}{3}x^2$ (PINK)

$y = 2x^2$ (BLACK)

$y = (x+3)^2$ (PURPLE)

$y = (x-3)^2$ (BLUE)

$y = x^2 + 2$ (GREEN)

$y = x^2 - 4$ (YELLOW)

How do you think this formula could be used in real life applications?

A real life example of quadratics and a parabola would be a bridge. This formula could help to measure bridges, etc. With the values of *a, h* and *k*, it would be possible to determine characteristics such as the line of symmetry (the centre/middle of the bridge) or the vertex (which would be the height of the bridge).

How do you know your generalizations about transformations are correct?

Justify/validate your general rules.

I know that my generalizations are correct based on my observations of how the parabola on the graph changes, just as the equation changes. For example, when observing the parabolas on the graph above, the red, dark blue and cyan parabolas all have their vertexes on the x-axis. By observing the equations, I am able to see that it is the value of the *h* coordinate which has an effect on how the parabola changes on the graph.

Can these generalizations be applied to other functions? Explain your reasoning.

I don't believe that this generalization can be applied to other functions. I looked at other quadratic equations that had the particular *a*, h and *k* coefficients..